

Quintom Cosmology with General Potentials

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Abstract

We investigate the phase-space structure of the quintom paradigm in the framework of a spatially flat, open, or closed isotropic and homogeneous universe. We examine the dynamical evolution under the assumption of late-time dark energy domination, without specifying the explicit quintom potential form. The obtained cosmological behavior is qualitatively different than that acquired from the single phantom model.

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1 Introduction

One of the most important problems of cosmology, is that of dark energy. Type Ia supernova observations suggest that the universe is dominated by dark energy with negative pressure, which provides the dynamical mechanism for the accelerating expansion of the universe [1, 2, 3]. The strength of this acceleration is presently a matter of debate, mainly because it depends on the theoretical model implied when interpreting the data. Most of these paradigms are based on the dynamics of a scalar or multi-scalar fields (e.g quintessence [4, 5] and quintom [6, 7] models of dark energy, respectively). In addition, other proposals on dark energy include interacting dark energy models [8], braneworld models [9], Chaplygin gas models [10], holographic dark energy [11], bulk holographic dark energy [12] and many others.

Initially, a scalar field candidate for dark energy was the quintessence scenario, which is based on a fluid with equation-of-state parameter lying in the range, $-1 < w < -1/3$. However, although investigations using many other models lead also to universe-acceleration below the de Sitter value [13], it is certainly true that the body of observational data allows for a wide parameter space compatible with an acceleration larger than this divide [14, 15]. If eventually this turn out to be the case, then the fluid driving the expansion would violate not only the strong energy condition $\rho + 3P > 0$, but the dominant energy condition $\rho + P > 0$, too. Fluids (fields) of such behavior are dubbed phantom fluids [16]. In spite of the fact that the theory of phantom fields encounter the problem of stability, which one could try to bypass by assuming them to be effective fields [17, 18], it is nevertheless interesting to study their cosmological implications and there are many relevant studies recently on phantom energy [19].

Although observations mildly favor dark energy models where w has crossed -1 in the near past, neither quintessence nor phantom can fulfill this transition. In quintessence model the equation of state $w = p/\rho$ is always in the range $-1 \leq w \leq 1$ for $V(\phi) > 0$. Meanwhile, in the phantom scenario, which comparing to the quintessence has the opposite sign in the kinetic term in the Lagrangian, one always obtains $w \leq -1$. Therefore, neither of these two models alone can fulfill the transition from $w > -1$ to $w < -1$ and vice versa. Furthermore, although in k-essence [20] one can have both $w \geq -1$ and $w < -1$, it has been lately shown in [21, 22] that the corresponding crossing is very unlikely to be realized during the evolution. However, one can show [6, 7] that considering the combination of quintessence and phantom in a qualitatively new model, the -1 -transition can be fulfilled, as can be clearly seen in [23]. This model, dubbed quintom, can produce a better fit to observational data than the more conventional paradigms. Finally, note that in the recent work [24] the authors prove in full generality the no-go theorem that in order to acquire the $w = -1$ crossing it is necessary to have more than one degrees of freedom. This theorem offers a concrete theoretical justification for the quintom paradigm.

In the literature, there has been a general investigation of the phase-space of a spatially flat homogeneous and isotropic universe dominated by a phantom field. In [25], the peculiar dynamics arising from a negative kinetic energy density has been studied with the help of a toy model consisting of two coupled oscillators, one with negative kinetic energy representing the phantom field ϕ , and the other with positive kinetic energy mimicking the gravitational field. The spacetime geometry is that of a spatially flat Friedmann-Lemaitre-Robertson-Walker (FLRW) universe, the assumed potentials maintain a general form, and the asymptotic dynamics (late-time attractors) is examined in

the three-dimensional phase-space $(H, \phi, \dot{\phi})$, under the condition of the final domination of the phantom. On the other hand, in [26] the authors have examined several scenarios which exhibit a late-time behavior where cold matter with zero pressure dominates the evolution and the phantom energy decays.

In the present study we desire to perform the phase-space investigation in the case of quintom cosmology, constructed using both phantom (ϕ) and quintessence (σ) fields. As was mentioned above, despite its outward structural similarity, quintom behavior is conceptually and qualitatively different from the phantom one. We desire to maintain a non-specific potential form $V(\phi, \sigma)$, and we assume that quintom dark energy eventually dominates the cosmic dynamics. Considering a non-spatially flat FLRW universe as the underlying spacetime geometry, we study quintom dynamics and we examine whether the universe decomposes in a singularity or expands forever.

2 Quintom cosmology

We are interesting in investigating the late-time cosmological evolution of the quintom field, consisting of the normal scalar field σ and the negative-kinetic-energy scalar field ϕ . We consider a general Friedmann-Lemaitre-Robertson-Walker universe with line element

$$ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) \quad (1)$$

in comoving coordinates (t, r, θ, φ) , where a is the scale factor and k denotes the space curvature with $k = 0, 1, -1$ corresponding to a flat, closed or open universe respectively.

The action which describes the quintom model is expressed in the following form [27, 28]:

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma + V(\phi, \sigma) \right], \quad (2)$$

where we have neglected the Lagrangian density of matter fields consistently to their late-time downgrading. The effective energy density ρ and the effective pressure P of the scalar fields, are given by [6]:

$$\rho = -\frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \dot{\sigma}^2 + V(\phi, \sigma) \quad (3)$$

$$P = -\frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \dot{\sigma}^2 - V(\phi, \sigma), \quad (4)$$

where the negative sign of ϕ -kinetic energy is characteristic of a phantom field.

The first Einstein field equation is:

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3M_p^2} \rho, \quad (5)$$

where H is the Hubble parameter defined, as usual, as $H \equiv \dot{a}/a$. Equation (5) can be considered as a constraint for the Hubble rate. Setting $8\pi/M_p^2 = 1$ and using (3), it is rewritten as:

$$H^2 + \frac{k}{a^2} = \frac{1}{6} [-\dot{\phi}^2 + \dot{\sigma}^2 + 2V(\phi, \sigma)]. \quad (6)$$

The other FLRW equation is:

$$\dot{H} = \frac{k}{a^2} + \frac{1}{2} (\dot{\phi}^2 - \dot{\sigma}^2). \quad (7)$$

Furthermore, the evolution equations for the two scalar fields in FLRW framework have the following form:

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\partial V}{\partial \phi} = 0 \quad (8)$$

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{\partial V}{\partial \sigma} = 0, \quad (9)$$

where dots denote differentiation with respect to the comoving time t . Note also that, as usual, only three of equations (6)-(9) are independent. Lastly, we mention that the solutions of (6)-(9) must lead to a non-negative total quintom energy density (given in (3)).

Finally, it will be helpful to notice that alternatively we can derive the field equations from the Lagrangian:

$$\mathbf{L} = a^3(\rho - P) = 3a\dot{a}^2 - 3ak + a^3 \left[\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\dot{\sigma}^2 + V(\phi, \sigma) \right] \quad (10)$$

or, equivalently from the Hamiltonian:

$$\mathbf{H} = 3a^3 \left[H^2 - \frac{k}{a^2} + \frac{1}{6}(\dot{\phi}^2 - \dot{\sigma}^2) - \frac{1}{3}V(\phi, \sigma) \right]. \quad (11)$$

3 The phase-space of the quintom paradigm

In the previous section we formulated the dynamical equations for the quintom model. In the present section we are interested in investigating its phase-space behavior for the various curvature cases.

It is obvious that in a spatially-flat FLRW universe (6) corresponds to $\mathbf{H} = 0$. Thus, for $k = 0$ equation (6) constitutes a Hamiltonian constraint which in the phase-space confines the orbits of the dynamical solutions of equations (6)-(9) to the surface of constant energy $\mathbf{H} = 0$ [25]. However, for $k \neq 0$ we obtain $\mathbf{H} \neq 0$ and in particular $\mathbf{H} = -6ak$. Thus, in a spatially non-flat universe this relation provides at each time a surface of constant energy in the phase-space, for orbits of the solutions of equations (6)-(9), and this surface is in general curved. In the following, instead of the scale factor, we choose the Hubble parameter to be a dynamical variable (since this is indeed the observable quantity) and thus the phase-space is five-dimensional: $(H, \phi, \dot{\phi}, \sigma, \dot{\sigma})$.

When the values of H , ϕ , σ and $\dot{\sigma}$ are specified, we can exert (6) in order to deduce the values of $\dot{\phi}$:

$$\dot{\phi}^2 = \dot{\sigma}^2 + 2V - 6 \left(H^2 + \frac{k}{a^2} \right). \quad (12)$$

This relation determines the sub-space of the phase-space that is forbidden for the dynamics. Indeed, defining the quantity

$$Q = \dot{\sigma}^2 + 2V - 6 \left(H^2 + k/a^2 \right), \quad (13)$$

(12) implies that the forbidden sub-space \mathcal{F} is specified as:

$$\mathcal{F} \equiv \{(H, \phi, \dot{\phi}, \sigma, \dot{\sigma}) : Q < 0\}. \quad (14)$$

Furthermore, in order to examine the properties of the allowed sub-space, we deduce from (12) that:

$$\dot{\phi} = \pm \sqrt{Q}. \quad (15)$$

Thus, for every H , ϕ , σ and $\dot{\sigma}$, there are two distinct values of $\dot{\phi}$. Therefore, we conclude that the sub-space of the five-dimensional phase-space that is accessible by the dynamics, consists of two four-dimensional sub-surfaces, one for each solution branch of (15). We call sub-surface \mathcal{A} the one corresponding to the positive branch, and \mathcal{B} the one arising from the negative branch. These two sub-surfaces meet at the three-dimensional sub-surface \mathcal{C} , which is defined as:

$$\mathcal{C} \equiv \left\{ (H, \phi, \dot{\phi}, \sigma, \dot{\sigma}) : V(\phi, \sigma) = 3 \left(H^2 + \frac{k}{a^2} \right) - \frac{\dot{\sigma}^2}{2}, \quad \dot{\phi} = 0 \right\}, \quad (16)$$

which belongs also to the boundary of the forbidden region \mathcal{F} and lies in the $(H, \phi, \sigma, \dot{\sigma})$ plane.

Let us now examine the fixed points of the system (6)-(9), which as usual are quested using the conditions: $\dot{\phi} = 0$, $\dot{\sigma} = 0$ and $\dot{H} = 0$. For the flat universe case ($k = 0$) these points (H_0, ϕ_0, σ_0) are easily extracted and they are determined by:

$$H_0 = \pm \sqrt{\frac{V_0}{3}}, \quad \frac{\partial V}{\partial \phi}|_0 = 0, \quad \frac{\partial V}{\partial \sigma}|_0 = 0, \quad (17)$$

where $V_0 \equiv V(\phi_0, \sigma_0)$ and the derivatives are calculated at this potential point. Thus, for the flat universe the equilibrium points for the dynamics are simply the de Sitter spaces with constant scalar fields. Note however that in order for these points to exist, the corresponding cosmological potentials must have points of zero gradient in both ϕ and σ directions. Furthermore, from (16) and (17) we deduce that the fixed points lie on the sub-space \mathcal{C} .

Let us make here a comment on the two distinct solutions of relation (17). As it was shown in [29], there is a cosmic duality between different quintom solutions. In particular, an expanding universe, which at early times is dominated by quintessence ($w > -1$) and lately by phantom ($w < -1$) fields, is dual to a contracting universe which faces the transition from $w < -1$ to $w > -1$. It is easy to see that the two distinct fixed-points given in (17), correspond to such expanding and contracting universes. This property is helpful for the investigation of the corresponding cosmological evolution.

For the $k \neq 0$ case the dynamical system (6)-(9) does not have fixed points, apart from the two extreme solutions, one with $H = 0$, $a \rightarrow \infty$, $\dot{a} < \infty$, $V_0 = 0$, and V 's derivatives equal to zero, and the other with $a \rightarrow \infty$, $H = \pm \sqrt{V_0/3}$, and V 's derivatives equal to zero. Thus, in the non-flat universe our model is always out of equilibrium. In this case it is useful to take advantage of the aforementioned duality between the different quintom solutions [29]. Namely, as we can easily see, an eternally expanding universe starting with an initial singularity at $t = 0^+$ is dual to a contracting one that begins with an infinite scale factor. In addition, a contracting universe ending in a Big Crunch at $t = 0^-$, is dual to an expanding universe ending in a final Big Rip at $t = 0^-$. This behavior is definitely qualitatively different from the flat paradigm. Obviously, the realization of a particular branch depends on the initial conditions.

It would be interesting to examine the case of an orbit with initial conditions chosen exactly on \mathcal{C} . We generalize [25] and we construct the tangent to the orbits as:

$$\vec{T} \equiv (\dot{H}, \dot{\phi}, \ddot{\phi}, \dot{\sigma}, \ddot{\sigma}) \equiv \left(\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\dot{\sigma}^2, \dot{\phi}, \frac{\partial V}{\partial \phi} - 3H\dot{\phi}, \dot{\sigma}, -\frac{\partial V}{\partial \sigma} - 3H\dot{\sigma} \right), \quad (18)$$

in the $(H, \phi, \dot{\phi}, \sigma, \dot{\sigma})$ space. On the boundary \mathcal{C} we have $\dot{\phi} = 0$ and thus the subsequent dynamics will depend on $\dot{\sigma}$ and on the potential derivatives. If

additionally $\dot{\sigma} = 0$, then $\vec{T}_C = (0, 0, \frac{\partial V}{\partial \phi}, 0, -\frac{\partial V}{\partial \sigma})$. Therefore, if $\partial V/\partial \phi|_C > 0$ and $\partial V/\partial \sigma|_C < 0$, an orbit beginning on \mathcal{C} will move into sub-surface \mathcal{A} , whereas if $\partial V/\partial \phi|_C < 0$ and $\partial V/\partial \sigma|_C > 0$ it will move inside sub-surface \mathcal{B} . For the crossed cases, we can not predict the specific evolution unless we know the explicit potential gradients. Note that these results hold for both flat and non-flat universe, in a unified way.

Finally, we are interesting in investigating the evolution of the Hubble parameter. In the case of a single phantom field in a flat universe background, it was shown in [25] that $\dot{H} > 0$ everywhere apart from the fixed points, and this behavior corresponds to super-acceleration [30]. However, in the present quintom model, H -behavior is different. Indeed, for all k cases the sign of \dot{H} , given by (7), can be varying and moreover (according to the values of $\dot{\phi}$ and $\dot{\sigma}$ at each moment) it can vary between the different stages of a specific universe evolution. Moreover, since H is not evolving monotonically, periodic orbits cannot be excluded a priori, and indeed there could be potentials leading to such a behavior, or even to cyclic universes [31].

Although in the aforementioned analysis, in order to examine the general evolution characteristics, we have remained in the general potential case, let us finish this section by giving a specific example using a simple potential form. We consider the quadratic potential $V(\phi, \sigma) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\sigma^2\sigma^2 + c_p$, where m_ϕ is the quintessence component mass, m_σ is the phantom component mass and c_p is a constant. In this case, for the flat universe case the corresponding fixed-points are the loci where $\phi = \sigma = 0$ and $H_0 = \pm\sqrt{c_p/3}$. Thus, for $c_p \neq 0$ we obtain de Sitter universes, while for $c_p = 0$ we obtain Minkowski spaces. Knowing the specific potential form it is straightforward to perform a stability analysis and show that these fixed-points are unstable. These results are in agreement with [29, 28]. Note also the duality between expanding and contracting universes (the two distinct H_0 cases) described above. In addition, for this quadratic case and imposing $\dot{\sigma} = 0$, we obtain $\vec{T}_C = (0, 0, m_\phi^2\phi^2, 0, -m_\sigma^2\sigma^2)$, leading to the result that an orbit beginning on \mathcal{C} will move into sub-surface \mathcal{A} .

4 Conclusions

In this work we have investigated the phase-space structure of the quintom paradigm without determining a specific potential form. Furthermore, our analysis was performed in a general FLRW geometrical background, covering the cases of a flat, open and closed universe. The quintom scenario is realized by a dynamical system consisting of two scalar fields, one with negative and the other with positive kinetic energy, where we assume that at late times the quintom fields dominate the dynamics. Our results are qualitatively different comparing to the single phantom model examined in [25], thus offering an additional argument for the conceptual difference between quintom and phantom cosmology.

The phase-space of the quintom model proves to consist of two connected four-dimensional sub-surfaces in the $(H, \phi, \dot{\phi}, \sigma, \dot{\sigma})$ space. In the flat universe framework, and provided that the potential $V(\phi, \sigma)$ possesses points of zero gradient in both ϕ and σ directions, the equilibrium solutions of the dynamics are just de Sitter spaces. In such cases the expansion may last for ever. However, in non-flat universes the behavior can be qualitatively different, since the dynamical motion is at all times out of equilibrium, and the universe is expanding forever or it results to a Big Rip.

Our investigation reveals that the Hubble parameter is a non-monotonic

function of time, and this is a radical difference comparing to the single phantom model [25]. Indeed, H can be increasing or decreasing in subsequent evolution stages of a specific universe. Furthermore, there could be special potential cases leading to periodic behavior, i.e to cyclic universes.

Quintom scenario offers an efficient explanation of the -1 crossing of the equation-of-state parameter of dark energy, thus being consistent with observations. From our investigations it is implied that it additionally offers a significantly larger variety of cosmological evolutions than the simple phantom model. These characteristics make quintom cosmology an interesting subject for further investigation.

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